FST 2-6 Notes
Topic: Quadratic Models
GOAL
Review the general quadratic equation $y = ax^2 + bx + c$, its graph, and the Quadratic Formula. Review or introduce quadratic modeling and use technology to determine the best fitting parabola.

SPUR Objectives

E Describe properties of quadratic functions.

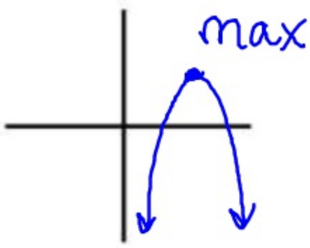
F Find and interpret quadratic regression and models.

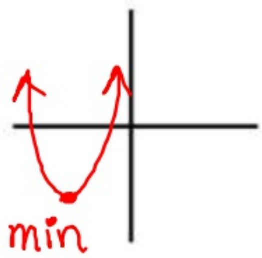
Vocabulary

quadratic model
quadratic regression

Properties of Quadratic Functions:

- $\overset{y}{f(x)} = ax^2 + bx + c$, where $a \neq 0$
- Graphs a parabola -U shaped
- Domain: $\{x \mid x \in \mathbb{R}\}$
- (0, c) is the y-intercept where graph crosses y-axis





If a < 0: a is negative

Opens down

Maximum point

If a > 0: a is positive

Opens up

minimum point

Range: $\{y \mid y \le \text{maximum value}\}$

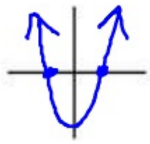
Range: $\{y \mid y \ge$ ~~maximum~~ minimum $\text{value}\}$

x-intercepts of quadratic equation:

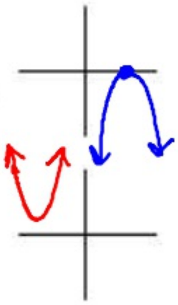
QUADRATIC FORMULA: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant, $b^2 - 4ac$, tells us information about the x-intercepts: — where graph hits the x-axis

If $b^2 - 4ac > 0$, there are 2 distinct x-intercepts.



If $b^2 - 4ac = 0$, there is only 1 x-intercept.



If $b^2 - 4ac < 0$, there are no x-intercepts.

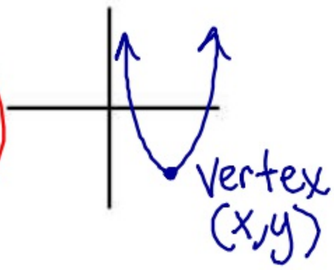
(x,y)

The vertex of the parabola is found by:

1) finding the x-coordinate, $x = -\frac{b}{2a}$ or $\frac{-b}{2a}$

2) using x from step 1 to find the y-coordinate, $y = ax^2 + bx + c$

$$\left(\frac{-b}{2a}, a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c\right)$$



ax^2+bx+c

1. Consider the function f with equation $\cancel{f(x)}\ y = -3x^2 - 4x + 7$.
   a. Find the x- and y-intercepts of its graph.
   b. Tell whether the parabola has a maximum or minimum point, and find its coordinates.

a) x-intercept, when $y=0$

$$0=-3x^2-4x+7$$

$a=-3 \quad b=-4 \quad c=7$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{4\pm\sqrt{(-4)^2-4(-3)(7)}}{2(-3)}$$

$$x=\frac{4\pm\sqrt{100}}{-6}$$

$\frac{4+\sqrt{100}}{-6}=\boxed{-2.\overline{3}}$

$\frac{4-\sqrt{100}}{-6}=1$

a) y-intercept (when $x=0$)

$$y=-3x^2-4x+7$$

$$y=-3(0)^2-4(0)+7$$

$y=7$ $\boxed{(0,7)\text{ y-int}}$

b) $a=-3$; opens down

Maximum pt

$$x=\frac{-b}{2a} \quad \frac{-(-4)}{2(-3)}=\frac{4}{-6}$$

$$\boxed{\left(\frac{-4}{6},\ 8\tfrac{1}{3}\right)}$$

$$y=-3x^2-4x+7$$

$$-3\left(\tfrac{-4}{6}\right)^2-4\left(\tfrac{-4}{6}\right)+7$$

$$y=8.\overline{3}$$

2. A projectile is shot from a tower 10 feet high with an upward velocity of 100 feet per second.
   a. Approximate the relationship between height h (in feet) and time t (in seconds) after the projectile is shot.
   b. How long will the projectile be in the air?

Height h of an object at time t after it has been thrown with initial velocity v_0 from an initial height h_0, $g = 32\,ft/\sec^2$

$$h = -\frac{1}{2}gt^2 + v_0 t + h_0$$

a) $h = -\frac{1}{2}(32)t^2 + 100t + 10$

$$\boxed{h = -16t^2 + 100t + 10}$$

b) $0 = -16t^2 + 100t + 10$

$a = -16 \quad b = 100 \quad c = 10$

$$\frac{-100 \pm \sqrt{(100)^2 - 4(-16)(10)}}{2(-16)}$$

$$\frac{-100 \pm \sqrt{10{,}640}}{-32}$$

$$\frac{-100 + \sqrt{10640}}{-32} \qquad \frac{-100 - \sqrt{10640}}{-32}$$

$\cancel{-.098 \text{ sec}}$ $\qquad$ 6.348 sec

3. A parabola contains the points (-0.1, -16.32), (2, 3), and (6, -9). Find its equation.

x y x y x y

By solving a system of equations: $y = ax^2 + bx + c$

Set up the System of Equation

$$-16.32 = a(-0.1)^2 + b(-0.1) + c$$

$$3 = a(2)^2 + b(2) + c$$

$$-9 = a(6)^2 + b(6) + c$$

Write a Matrix Equation

$$\begin{bmatrix} .01 & -0.1 & 1 \\ 4 & 2 & 1 \\ 36 & 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -16.32 \\ 3 \\ -9 \end{bmatrix}$$

Write the equation for the parabola

$$y = -2x^2 + 13x - 15$$

L1	L2
X	Y
-0.1	-16.32
2	3
6	-9

By using quadratic regression

STAT #1 Edit Data x values in L1 y values in L2

STAT $\rightarrow$ CALC #5

**When you identify you're a, b and c values don't forget to substitute them back into the equation $y = ax^2 + bx + c$